DOCUMENT RESUME

ED 460 015 SE 065 600

AUTHOR	Young, Anne Ludington
TITLE	A Calculus I Project: Discovering the Derivative of an Exponential Function.
PUB DATE	1996-06-00
NOTE	6p.; Paper presented at the Annual Conference on the Teaching of Mathematics (5th, Baltimore, MD, June 21-22, 1996). In: Winkel, Brian, Ed. "PRIMUS: Problems, Resources, and Issues in Undergraduate Mathematics Studies" v7 n1 Mar 1997.
AVAILABLE FROM	PRIMUS, Department of Mathematical Sciences, United States Military Academy, West Point, NY 10996 (Back/Single issues, $12; Annual Subscription, $44). For full text: http://archives.math.utk.edu/CTM/FIFTH/Young/paper.pdf.
PUB TYPE	Journal Articles (080) -- Reports - Descriptive (141) -- Speeches/Meeting Papers (150)
JOURNAL CIT	PRIMUS; v7 n1 Mar 1997
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Calculus; Higher Education; *Mathematics Activities; *Mathematics Instruction; Secondary Education

ABSTRACT

This paper describes a project one teacher assigns to Calculus I students at the Department of Mathematical Sciences at Loyola College in Baltimore, Maryland. The project is introduced to the students early in the semester and requires the use of technology, team work, and the completion of an extensive writing component. The goal of the project is to lead the students to the formula for the derivative of an exponential function. The teacher assigns the project after the class has studied the meaning of the derivative and is familiar with estimating it graphically and numerically, but before any algebraic formulas are covered. The project is divided into three parts. In the first part, students are asked to estimate numerically the derivatives of four exponential functions. In the second part, they are to write a letter to a classmate explaining their solutions to Part 1. Finally, in Part 3 the students compose a newsletter article summarizing their conclusions and/or conjectures. This project is unique in that it not only engages for the students but also motivates them to consider the subject matter from a new and different perspective by giving them the opportunity to participate in the exciting mathematical process of making discoveries and conjectures. Student version of project overview and instructions is appended. (MA)

Reproductions supplied by EDRS are the best that can be made
from the original document.

ED 460 015

A Calculus I Project : Discovering the Derivative of an Exponential Function

Anne Ludington Young
Department of Mathematical Sciences
Loyola College in Maryland
Baltimore MD 21210

Abstract: Numerical estimates of the derivative of a^x are plotted. Exponential curve fitting techniques yield the algebraic formula. This basic project leads to extra credit problems.

I teach a reformed, scientific calculus course using the Hughes-Hallet text. As in many such courses, I assign several projects during the semester. These projects require students to work in teams of two or three. Projects are longer and more complicated than homework problems or routine laboratory exercises. Like almost all work in the course, projects require the use of technology. In addition, they have a significant writing component.

The first project that I assign in Calculus I leads to the formula for the derivative of an exponential function. It gives students the opportunity to participate in the exciting mathematical process of making discoveries and conjectures. I assign the project after completing Chapter 2. At this point students have studied the meaning of the derivative and know how to estimate it graphically and numerically. They have not yet learned any algebraic formulas.

Since the majority of the class are first year students and this is their first project, I divide it into three parts. In Part I students numerically estimate the derivatives of four exponential functions; e.g., 1.83^x, 2.83^x, 3.83^x and 4.83^x. In Part II they write a letter to a classmate explaining their solutions to Part I. Finally, in Part III they write a newsletter article summarizing their conclusions and/or conjectures. The actual project as I distribute it to my students appears at the end of this paper.

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY
A. Young
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

BEST COPY AVAILABLE

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
X This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.
• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

The project allows students to apply and integrate several different topics that they have studied in Chapters 1 and 2. From Chapter 2 they know how to estimate numerically the derivative at a point. By repeating this process at several points, they generate a table for the derivative function. In Chapter 1 students learned how to fit a formula to a table and/or a graph. In particular they studied this process for an exponential function. Part I of the project requires putting these steps together. This is surprisingly difficult for some students.

I have students turn-in their answers to Part 1 before they proceed to the other two parts. This allows me to point out mistakes and/or make comments. For example, usually one group will try to fit different types of functions to the derivatives of the various functions; e.g., a line to the first, a quadratic to the second and an exponential to the third. Other groups may use the e^{ax} form of an exponential function; in this case, I note the alternative form. Still other students may approximate the derivative of 1.83^x with something like $c \cdot 1.8299^x$. In this case, I point out that rounded to two decimal places the derivative is $c \cdot 1.83^x$.

Part II of the project gives the students a targeted writing assignment. They must write a letter to a sick classmate explaining their solutions to Part I. I penalize students if they do not write a letter or if they fail to include wishes for a quick recovery. Most teams are successful with this part of the assignment and students seem to enjoy it.

In Part III students must write a newsletter article for their classmates summarizing the team's conclusions and/or conjectures. What do I expect/hope that students will include in their articles? First, that the derivative of an exponential function seems to an exponential function. Second, that the base seems to remain the same. Third, if $D(b^x) = k \cdot b^x$, then $k = f'(0)$. Most teams do not make all of these observations. In fact, many do not even make the first. Because they do not understand what it means to draw conclusions from their observations, some groups end up repeating their letters.

It is perhaps not surprising that students have a great deal of difficulty with Part III. Students have little or no experience making mathematical conjectures. Even when they have written the derivatives of 1.83^x and 2.83^x as $k_1 \cdot 1.83^x$ and $k_2 \cdot 2.83^x$, respectively, they are unable to conjecture that the derivative of b^x is $k \cdot b^x$.

The project leads to several additional problems which I offer as extra credit possibilities. Each of these problems is based on the assumption that the derivative of $f(x) = b^x$ is $f'(x) = f'(0) \cdot b^x$. These problems as I distribute them to my students appear at the end of this paper.

In the first problem, students are to estimate the number B for which $D(B^x) = B^x$. Of course, the result is an estimate for e. In the second, they are to estimate k(b) where $D(b^x) = k(b) \cdot b^x$. This leads to the formula $D(b^x) = \ln b \cdot b^x$. Finally, in the third problem, students are to use the limit definition of derivative to algebraically derive $D(b^x) = f'(0) \cdot b^x$.

If students correctly solve any of these additional problems, I distribute their newsletters to the entire class. In any case, I discuss my solutions to all three problems. They are the basis of my discussion for the algebraic approach to the exponential function.

STUDENT VERSION OF THE PROJECT

Part I:

You and your partner have been assigned a 2 digit number, gh. Use gh to form a new number, 1.gh. For example, if your number is 83, then your new number is 1.83.

Now consider the function $f(x) = (1.gh)^x$. (In the example above, the function would be $f(x) = (1.83)^x$.) Estimate the derivative of f(x) at each of the following points:

$$x = -1.5, -1, -0.5, 0, 0.5, 1, 1.5.$$

Record your answers on the worksheet. When you have finished, you will have generated a table. Plot these values. Find a formula that fits the data in the table. Add the graph of the formula to your plot.

Now, repeat the above directions for 3 other functions:

$$f(x) = (2.gh)^x \text{ , } f(x) = (3.gh)^x \text{ and } f(x) = (4.gh)^x \text{ .}$$

When you have finished, turn in your worksheets and accompanying graphs. I will grade them, verifying correct answers and indicating mistakes. These worksheets are due no later than day d. (The project was distributed on day d – 5.)

Part II:

Susan is a Loyola student who is taking Calculus I. Shortly after her class completed Section 2.3, Susan went skiing for the weekend. Unfortunately, she was in an accident and broke her leg in several places. Although she is recovering, she is currently in traction in the hospital. Happily, she is able to resume studying calculus.

Write a letter to Susan describing your solution for $f(x) = (1.gh)^x$. That is, tell her how you estimated the values in the table and how you found the formula. Susan's been through a lot lately, so she's a little fuzzy on calculus. You should tell her not only what you did, but also remind her about why you did it.

Susan, of course, does not have access to any computer software in the hospital. She does however have her graphing calculator. In your letter, give her enough information so that she can reproduce your results and then do $f(x) = (2.gh)^x$ on her own.

Your letter to Susan should be no more than 2 pages. It must be typed; complicated equations and the like can be neatly handwritten. Your letter is due on day d + 5.

Part III:

This semester we are going to publish our own in-class newsletter. In newsletter articles, students will report their discoveries. At the end of an article, students may make conjectures and/or ask questions. Newsletter articles must meet the following criteria:

1. Articles must be typed. However, complicated equations can be neatly handwritten.
2. Graphs/tables can appear in the body of the article or at the end. They must be clearly labeled.

3. Articles must be at least half a page and no more than 2 pages in length. This page limit includes any accompanying graphs/tables.

Look over your worksheets. Use the results to write a newsletter article. Your article is due on day d + 7.

STUDENT VERSION OF EXTRA CREDIT PROBLEMS

Based on your work for Project 1, it appears that the derivative of an exponential function is an exponential function with the same base. That is, the derivative of $f(x) = b^x$ is $f'(x) = k \cdot b^x$. Now if we substitute $x = 0$, we get

$f'(0) = k \cdot b^0 = k$.

Thus, it appears that the derivative of $f(x) = b^x$ is $f'(x) = f'(0) \cdot b^x$.

Here are 3 additional problems. For extra credit, do one of them and summarize your results/conclusions in a newsletter style article.

1. Assume that the derivative of $f(x) = b^x$ is $f'(x) = f'(0) \cdot b^x$. Thus we might say that the "nicest" base for an exponential function is the number B for which $f'(0) = 1$. Find an estimate for that particular base B. That is, estimate the number B for which the derivative of $f(x) = B^x$ is $f'(x) = B^x$.

2. Assume that the derivative of $f(x) = b^x$ is $f'(x) = k \cdot b^x$. The constant k depends on b. Thus, we could write $f'(x) = k(b) \cdot b^x$. As mentioned above, $k = k(b) = f'(0)$. Now, for $f(x) = b^x$,

$$k(b) = f'(0) \approx (b^{0.01} - 1)/0.01.$$

Estimate $k(b) = f'(0)$ for different values of b. That is, generate a table containing b and k(b). Plot the points of the table and try to fit a formula to the table. Then use your formula for k(b) to find a formula for the derivative of $f(x) = b^x$.

3. For $f(x) = b^x$, write down the limit definitions of $f'(x)$ and $f'(0)$. Explain why algebraically it makes sense that $f'(x) = f'(0) \cdot b^x$.